Shorty Paper: Waldman-Beltrone Divsion

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1 Motivation and Standard Approach

Suppose you were looking to receive $1000 from a tax-deferred savings account with a tax rate of $t = 40\%$***. How much would you have to withdraw in order to end up with $1000?***

There are at least two approaches to this. To societies who've discovered division, we can simply set up equation (1), meaning "My post-tax remainder r of what total withdrawal x yields $1000?"

$$rx = 1000 \tag{1}$$
$$(1-t)x = 1000 \tag{2}$$
$$x = \frac{1000}{1-t} \tag{3}$$
$$\tag{4}$$

In this case, (3) show us we'd need to withdraw $\frac{1000}{1-.4} = 1666.67$ to get to 1000 after taxes.

However, there does exist another way to get this figure without the bother of conventional division.

2 Iterated Withdrawal

Considered by L. F. Waldman, possibly among others, the following method *also* produces the desired effect of getting the right amount of post-tax money out.

1. Start with your shortfall x_0. In the the above example $x_0 = 1000$.

2. Withdraw your shortfall x_i from the bank. After setting aside tax debt $x_i \cdot t$, set your new shortfall $x_{i+1} \leftarrow x_i \cdot t$.

3. If shortfall is less than the quantum unit of currency ϵ, end with your withdrawal and debt piles completed. Else set $i \leftarrow i + 1$ and go to step (1).

Thus, instead of withdrawing $1666.67 and setting aside $666.67 for taxes, we:

- **Withdraw 1000**, set aside 1000 * .4 = 400. Total post-tax: 600.
- **Withdraw 400**, set aside 400 * .4 = 160. Total post-tax: 600 + 240.
- **Withdraw 160**, set aside 160 * .4 = 64. Total post-tax: 600 + 240 + 96.
- **Withdraw 64**, set aside 64 * .4 = 25.60. Total Post tax = 600 + 240 + 96 + 38.4
- ...

After an infinite number of iterations, We end up with $1666.67 withdrawn from the bank, $1000 in our pocket and $666.67 for the tax man. Simple!

3 Proof

A pre-tax withdrawal of $\frac{x}{1-t}$ before application of tax at rate t produces post-tax income of x. This exercise is left as proof to Larry Waldman[1]. We prove that the Waldman-Beltrone method of withdrawal also produces this result.

If $|t| < 1$, the well known series $Q = 1 + t + t^2 + t^3...$ converges:

$$Q = 1 + t + t^2 + t^3 + ... = \sum_{r=0}^{\infty} t^r \quad (5)$$

$$tQ = t + t^2 + t^3 + t^4... = \sum_{r=0}^{\infty} t^r \quad (6)$$

$$(1 - t)Q = 1 \quad (7)$$

$$Q = \frac{1}{1 - t} \quad (8)$$

$$\quad (9)$$

But we see that Q is exactly what we're calculating in Waldman-Beltone wfithdrawal:

- **Withdraw shortfall of 1000** $= 1000 * 1 = xt^0$

[1] Also known as "the" reader.

- **Withdraw shortfall of 400** $= 1000 * .4. = xt^1$
- **Withdraw shortfall of 160** $= 1000 * .4 * .4 = xt^2$
- **Withdraw shortfall of 64** $= 1000 * .4 * .4 * .4 = xt^3$.
- ...

We can see that our total withdrawal ends up being $x(1 + t + t^2 + t^3 + ...) = x\frac{1}{1-t}$ as above.

4 So what?

This means that *we can compute division* $\frac{x}{d}$ *with* $-1 < d < 1$ *entirely from the operations of addition, subtraction, and multiplication.* We can easily expand this to add $d \neq 0$ with the addition of a simple [decimal] shift operator $shift(x, a)$ which shifts the decimal point a units left if $a \leq 0$ and a units right if $a > 0$. If we're operating in base b, this means multiplying by b^a.

Computing $\frac{x}{r}$ **via Waldman-Beltrone division to precision** ϵ:

1. Shift r by a places until $|r| < 1$.
2. $tot \leftarrow 1, i \leftarrow 0, t_0 = 1 - r$
3. $t_{i+1} \leftarrow t * t_i, tot \leftarrow t_{i+1}$
4. $i \leftarrow i + 1$
5. If $xt_i < \epsilon$, go to step 3.
6. Otherwise, shift $x * tot$ by a places, and return.

The advantages to W-B divison include:

- Ability to implement with only addition, subtraction (to get $t = 1-r$), multiplication, and shift operators.
- May impresses your friends.

The disadvantages include:

- Theoretically takes infinite time to complete.
- Difficult and absurd.

We also acknowledge that long division also only requires the operations of addition, subtraction, multiplication, and shift (and some sort of "compare"), but we are not personal friends with Mr. Long, nor do we care to be.